GRAPH THEORY

Tutorial – 4

1. Let $d_1 \leq d_2 \leq \ldots \leq d_n$ be the vertex degrees of a simple graph G. Prove that, G is connected if $d_j \geq j$ when $j \leq n-1-d_n$.

2. Let G be an n-vertex digraph with no cycles. Prove that, the vertices of G can be ordered as $v_1, \ldots, v_n$ so that if $v_i v_j \in E(G)$, then $i < j$.

3. Suppose that G is a graph and D is an orientation of G that is strongly connected. Prove that if G has an odd cycle, then D has an odd cycle.

4. For $n \geq 4$, prove that the minimum number of edges in an n-vertex graph with diameter 2 and maximum degree (n-2) is (2n-4).

5. Prove that a d-regular simple graph G has a decomposition into copies of $K_{1,d}$ if only if G is bipartite.